GCE Examinations

Advanced Subsidiary / Advanced Level

Further Pure Mathematics
Module FP1

Paper B

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks should be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



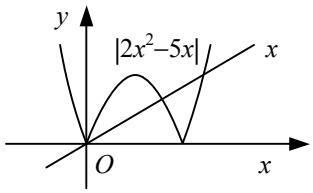
Written by Shaun Armstrong & Chris Huffer

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FP1 Paper B – Marking Guide

1.



B2

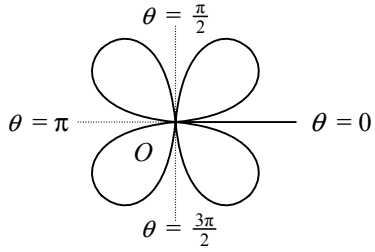
considering $2x^2 - 5x = x$ gives $2x(x-3) = 0 \ \therefore x = 0, 3$ — M1 A1

considering $-(2x^2 - 5x) = x$ gives $2x(x-2) = 0 \ \therefore x = 0, 2$ — A1

using graphs, require $2 < x < 3$ — A1 **(6)**

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2. *(a)*



B3

(b) $\frac{1}{2}\int_0^{\frac{\pi}{2}} a^2 \sin^2 2\theta \ \mathrm{d}\theta$ — M1

$= \frac{1}{4}a^2 \int_0^{\frac{\pi}{2}} 1 - \cos 4\theta \ \mathrm{d}\theta$ — M1

$= \frac{1}{4}a^2[\theta - \frac{1}{4}\sin 4\theta]_0^{\frac{\pi}{2}}$ — A1

giving $\frac{1}{8}a^2\pi$ — M1 A1 **(8)**

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3. *(a)* $= \sum_{r=1}^{n} (r^3 - r^2 + r - 1)$ — M1

$= \frac{1}{4}n^2(n+1)^2 - \frac{1}{6}n(n+1)(2n+1) + \frac{1}{2}n(n+1) - n$ — M1 A1

$= \frac{1}{12}n[3n(n^2 + 2n + 1) - 2(2n^2 + 3n + 1) + 6(n+1) - 12]$ — M1

$= \frac{1}{12}n[3n^3 + 2n^2 + 3n - 8]$ — A1

$= \frac{1}{12}n(n-1)(3n^2 + 5n + 8)$ — A1

(b) $\sum_{r=5}^{25} (r^2+1)(r-1) = \sum_{r=1}^{25} (r^2+1)(r-1) - \sum_{r=1}^{4} (r^2+1)(r-1)$ — M1

$= \frac{1}{12}.25.24.2008 - \frac{1}{12}.4.3.76$ — M1

$= 100\,400 - 76 = 100\,324$ — A1 **(9)**

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4. *(a)* int. fac. $= \mathrm{e}^{\int -\cot x \, \mathrm{d}x} = \mathrm{e}^{-\ln|\sin x|} = \operatorname{cosec} x$ — M1 A1

$\therefore \operatorname{cosec} x \frac{\mathrm{d}y}{\mathrm{d}x} - y \operatorname{cosec} x \cot x = \operatorname{cosec} x \times 2 \sin x \cos x$ — M1

$\frac{\mathrm{d}}{\mathrm{d}x}(y \operatorname{cosec} x) = 2\cos x$ — A1

$y \operatorname{cosec} x = \int 2\cos x \ \mathrm{d}x$

$y \operatorname{cosec} x = 2\sin x + c$ or $y = 2\sin^2 x + c \sin x$ — M1 A1

(b) $x = \frac{\pi}{6}, y = 2 \ \therefore c = 3$ — M1 A1

$\therefore$ when $x = \frac{2\pi}{3}$, $y = \frac{3}{2}(1 + \sqrt{3})$ — A1 **(9)**

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5. *(a)* f(1.0) = −0.0986; f(1.1) = 0.305
f cont. over interval, change of sign ∴ root — M1 A1

(b) $f'(x) = 3x^2 + \frac{2x}{4-x^2}$ — M1 A1

$x_{n+1} = x_n - \dfrac{x_n^3 - \ln(4-x_n^2)}{3x_n^2 + \frac{2x_n}{4-x_n^2}}$ — M1 A1

$x_0 = 1.0$, $x_1 = 1.026894$, $x_2 = 1.026221$ — A2

f(1.0262205) = -2.02×10^{-6}; f(1.0262215) = 1.83×10^{-6}
change of sign ∴ root ∴ α = 1.026221 correct to 6dp — M1 A1 **(10)**

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6. *(a)* $|z_1| = \sqrt{(49+1)} = \sqrt{50} = 5\sqrt{2}$ — B1
$5\sqrt{2} \times |z_3| = 50$ ∴ $|z_3| = 5\sqrt{2}$ — M1 A1

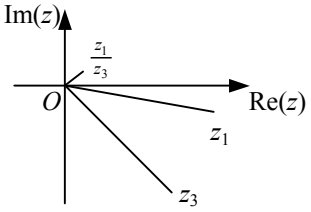
(b) $\arg z_2 = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$ — B1

$\frac{\pi}{3} - \arg z_3 = \frac{7\pi}{12}$ ∴ $\arg z_3 = -\frac{\pi}{4}$ — M1 A1

(c) $z_3 = 5\sqrt{2}[\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})] = 5 - 5i$ ∴ $a = 5$, $b = -5$ — M1 A1

(d) $\frac{z_1}{z_3} = \frac{7-i}{5-5i} \times \frac{5+5i}{5+5i} = \frac{40+30i}{50} = \frac{1}{5}(4+3i)$ — M2 A1

(e)



B2

(f) e.g. $|z_1| = |z_3|$ ∴ $|\frac{z_1}{z_3}| = |\frac{z_3}{z_1}|$; $\arg\frac{z_1}{z_3} = \arg z_1 - \arg z_3 = -\arg\frac{z_3}{z_1}$
same modulus, -ve of argument ∴ conjugate — M1 A1 **(15)**

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7. *(a)* $\frac{dx}{dt} = e^t$; $\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = e^{-t}\frac{dy}{dt}$ — M1 A1

$\frac{d^2y}{dx^2} = \frac{dt}{dx}(e^{-t}\frac{d^2y}{dt^2} - e^{-t}\frac{dy}{dt}) = e^{-2t}(\frac{d^2y}{dt^2} - \frac{dy}{dt})$ — M1 A2

(b) $e^{2t}.e^{-2t}(\frac{d^2y}{dt^2} - \frac{dy}{dt}) - e^t.e^{-t}.\frac{dy}{dt} - 3y = 6.e^{2t}$ — M1 A1

giving $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 3y = 6e^{2t}$ — A1

(c) aux. eqn. $m^2 - 2m - 3 = 0$ — M1
$(m+1)(m-3) = 0$; $m = -1, 3$ C.F. $y = Ae^{-t} + Be^{3t}$ — A1

for P.I. try $y = Ce^{2t}$ ∴ $\frac{dy}{dt} = 2Ce^{2t}$, $\frac{d^2y}{dt^2} = 4Ce^{2t}$ — M1

so $4Ce^{2t} - 4Ce^{2t} - 3Ce^{2t} = 6e^{2t}$ ∴ $C = -2$ — M1 A1

gen. soln. $y = Ae^{-t} + Be^{3t} - 2e^{2t} = \frac{A}{x} + Bx^3 - 2x^2$ — A1

$\frac{dy}{dx} = -\frac{A}{x^2} + 3Bx^2 - 4x$ — M1

$x = 1, y = 3$ ∴ $3 = A + B - 2$

$x = 1$, $\frac{dy}{dx} = -5$ ∴ $-5 = -A + 3B - 4$ giving $A = 4$, $B = 1$ — M1 A1

∴ $y = x^3 - 2x^2 + \frac{4}{x}$ — A1 **(18)**

Total **(75)**

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Performance Record – FP1 Paper B

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	Total
Topic(s)	inequality	polar coords	series	1st order diff. eqn	numerical methods	complex nos.	2nd order diff. eqn	
Marks	6	8	9	9	10	15	18	75
Student								

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